Determination of Earth gravity field spherical harmonic coefficients using SLR data

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Satellite Laser Ranging

MOBLAS6, SLR operates in global network, 200 mJ, 200 pico-second pulse, accuracy about 2 cm
Earth Oblateness and J2

Gravity potential is described by spherical harmonic expansion

\[ V = \frac{GM}{r} \sum_{n=2}^{l} \sum_{m=0}^{n} \left( \frac{a}{r} \right)^{n} (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) P_{nm}(\sin \phi) \]

For a rotational symmetric Earth

\[ J_{2} = -C_{20} = \frac{2}{3} f - \frac{1}{3} m_{s} + O(f^2) \]

\[ f = \frac{a - b}{a} = \frac{1}{298.257} \quad m_{s} = \frac{\omega^{2}a^{3}}{GM} \approx 0.00346 \approx 1.03f \]

\[ \Delta J_{2} \approx 0.323 \Delta f \approx -0.323 \frac{f}{a} \Delta a = -1.7 \times 10^{-10} (1/m) \cdot \Delta a(m) \]

\[ \Delta a(m) = -0.59 \times 10^{10} \cdot \Delta J_{2} \]

\[ \delta \omega \approx -30.331 \Delta J_{2} \ ("/sec) \]

\[ \Delta J_{2} = -\frac{1 + k_{2}'}{5} \frac{R^{2}}{M} \sum \Delta m(\phi_{i}) \overline{P}_{2}(\sin \phi_{i})S_{i} \]

\[ J_{2} = -2.6 \times 10^{-11} / year \]

- Variations in J2 can be calculated as the weighted sum of zonal net mass changes
- These variations represent the imbalance of zonal mass variations between ‘tropical’ and extratropical area.
Changes to the lower degree spherical harmonics of the static gravity field are a result of large scale mass redistribution of the Earth system.

- Seasonal and shorter period sources include air and water movement within the atmosphere, oceans, hydrosphere, and cryosphere.
- Long term sources include redistribution within the solid Earth due to tectonics and glacial isostatic adjustment.
- At periods less than a few years, air and water contributions dominate, providing global-scale measurement of climate processes (air, water, and ice mass redistribution) from geodetic observations.
SLR provides long time series

- SLR technique is sensitive enough to measure the lowest degree even zonal harmonics (especially -C20, also termed J2)
- Most SLR satellites are at high altitudes, (e.g., 5860 and 5620 km for LAGEOS I & II) making them less sensitive to changes at higher spherical harmonic degrees.
- However, their high altitude means slow orbital decay relative to lower altitude satellites, providing long time series
- GRACE, CHAMP etc. high resolution, shorter time series
Long time series applications

• SLR tracking to passive geodetic satellites valuable due to 25+ years of tracking e.g. LAGEOS (but does not have resolution of low Earth orbiters).

• LAGEOS has been used to detect seasonal changes and has contributed to studies of geopotential zonal rates.

• The literature presents variations of the zonal and lower order and degree harmonics in good agreement with geophysical models of surface mass redistribution.
Standard approach

• Standard approach to determine gravity field from satellite orbits (using SLR, GNSS, DORIS) utilises changes in the orbital parameters, Kaula’s Linear Satellite Theory (LST)

Satellite perturbations due to the geopotential are expressed in Kaula’s theory (Kaula, 1966) with reference to a secularly precessing ellipse defined at any instant of time $t$ by the classical set of elements $\{a_0, e_0, i_0, \omega_0 + \dot{\omega}(t - t_0), \Omega_0 + \dot{\Omega}(t - t_0), M_0 + \dot{M}(t - t_0)\}$. Here subscripts indicate quantities at epoch, while dotted variables are the constant secular rates due to the even zonal harmonics. The perturbed mean motion is defined as $\dot{M} = n + \dot{M}_0$, where $n = \sqrt{GM_e/a_0}$ is the Keplerian mean motion about a central body with gravitational parameter $GM_e$ and $\dot{M}_0$ is the secular rate of the mean anomaly.

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This approach

- Based on gradient of gravity, i.e. acceleration experienced by satellite
• J2, J3, J4, J5, is determined through solving for the acceleration in a least squares adjustment (J6-J20 consider parameters, C21+S21)

\[
\ddot{x}_{lm} = \left\{ \frac{GM}{R_\oplus} \cdot \{-C_{l0}V_{l+1,1} \}\right.
\]

\[
m > 0 \quad \left\{ \left. \frac{GM}{R_\oplus} \cdot \frac{1}{2} \right\} \begin{cases}
(\frac{-C_{lm}V_{l+1,m+1} - S_{lm}W_{l+1,m+1}}{l-m+1}) \\
+ \frac{(l-m+2)!}{(l-m)!} \cdot (\frac{+C_{lm}V_{l+1,m-1} + S_{lm}W_{l+1,m-1}}{l-m+1})
\end{cases}
\]

\[
\ddot{y}_{lm} = \left\{ \frac{GM}{R_\oplus} \cdot \{-C_{l0}W_{l+1,1} \}\right.
\]

\[
m > 0 \quad \left\{ \left. \frac{GM}{R_\oplus} \cdot \frac{1}{2} \right\} \begin{cases}
\frac{-C_{lm}W_{l+1,m+1} + S_{lm} \cdot V_{l+1,m+1}}{l-m+1} \\
+ \frac{(l-m+1)!}{(l-m)!} \cdot (\frac{-C_{lm}W_{l+1,m-1} + S_{lm}V_{l+1,m-1}}{l-m+1})
\end{cases}
\]

\[
\ddot{z}_{lm} = \left\{ \frac{GM}{R_\oplus} \cdot \{(l-m+1) \cdot (\frac{-C_{nm}V_{l+1,m} - S_{lm}W_{l+1,m}}{l-m+1})\}\right.
\]

• The partial derivatives are passed to the sensitivity matrix as part of the rigorous linearisation of the orbit trajectory, together with the various parameters that determine the various forces (e.g. gravitational attraction of the moon, sun and planets) affecting the satellite orbit.
Model assumptions

- The sensitivity of a satellite to gravitational perturbations depends partially on its inclination and altitude.
- Observations from a single satellite contain the total effects of the Earth’s gravity field and its temporal changes.
- Therefore satellites at various altitudes and inclinations must be used for separating and constraining the individual spherical harmonics of the Earth’s gravity field.
LAGEOS was Laser GEOdynamics Satellite-1 (LAGEOS) was designed by NASA and launched in 1976. It was the first spacecraft dedicated exclusively to high-precision laser ranging and provided the first opportunity to acquire laser-ranging data that were not degraded by errors originating in the satellite orbit or satellite array. LAGEOS-2, based on the LAGEOS-1 design, was built by the Italian Space Agency and was launched in 1992.

**LAGEOS Parameters**

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*Courtesy of ASI*
Etalon are a Russian family (Etalon-1, Etalon-2) of passive geodetic satellites dedicated to satellite laser ranging. Etalon-1 was the first geodynamic satellite launched by the former Soviet Union. The Etalon spacecraft were launched in 1989 in conjunction with a pair of GLObal'naya NAVigatsionnaya Sputnikovaya Sistema (GLONASS) satellites. The mission objectives were to determine a high accuracy terrestrial reference frame and earth rotation parameters, to improve the gravity field, and to improve the gravitational constant.
Five geodetic satellites were used in this study to derive time series of spherical harmonics coefficients: LAGEOS-1 and LAGEOS-2, but also ETALON-1, AJISAI, STARLETTE.

![Time series of $C_{2,0}$: combined solution](image)

**Fig. 1.** Time series for the $C_{2,0} = -J_2$ coefficient.

1. Deleflie, F. and Exertier, P. © Société Française d'Astronomie et d'Astrophysique (SF2A) 2008
Published results differ to some extent...

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**Figure 3.** (a) $J_o$ (unnormalized) from Lageos1 & 2 with annual and semi-annual fit (solid line). (b) $J_o$ from Lageos with annual and semi-annual periodicities removed with six month boxcar average (solid line).

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Inkaba yeAfrica Workshop 6 @ SAGA 11, Swaziland 16 - 18 September 2009
Preliminary results (being processed as we speak)

Scale is correct
Sensitivity is correct
Only one satellite (LAGEOS 1)
Some results

- Linear drift in J2 estimated using GRACE derived J2 as starting point
- Estimated J2 value at point in future (16 months later)
- Determined rate as $\dot{C}_{20} = 8.49016 \times 10^{-12} / \text{yr}$
- This is about 27% below the IERS rate $\dot{C}_{20} = 1.16276 \times 10^{-11} / \text{yr}$
- But is explained by the nonlinear drift rate at the period of investigation, which makes direct comparison with a long term IERS rate difficult.
Future work

• Include more satellites in a combined solution (Starlette, Lageos-2, Ajisai, Etalon 1 and 2), code to be developed
• Determine correlation coefficient between two estimated parameters (J2, J3,...) as this is the indication of their separation in a simultaneous solution
• Once the annual variation of J2 is well determined it should be compared with results from modelling the atmospheric and oceanic mass redistribution and changes in the surface water
• The disturbing effect and magnitude of unmodelled forces should be estimated (model residuals could be absorbed by the J2,...estimates during least squares adjustments)
• Need to get hold of fast, multi-core, built for speed PC, as processing time is currently a major obstacle (good news 16.9.2009, IyA to provide some funds for fast computer!)
• Christina Botai (PhD student) will utilise developed software to evaluate and compare different gravity models
• Need more students to work on SLR/LLR project
Thank you